GROWTH OF CRACKS WITH CREEP, TAKING ACCOUNT OF THE PLASTIC ZONE NEAR THE TIP OF A CRACK

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The process of the fracture of metals with creep as a function of the temperature of the stress takes place in a dual manner. Large stresses and a low temperature led to intergrain cracking with considerable irreversible deformations (viscous fracture). With small stresses and a high temperature, there is mainly the breaking of intergrain bonds with a small value of the irreversible deformation (brittle fracture). On a diagram of the long-term strength in the coordinates $\log \sigma_0 - \log t_R$, they are illustrated by two straight lines with different slopes.

Brittle fracture takes place by the generation of pores along the grain boundary, orthogonal to the direction of the maximal elongational stress, which, growing during the process of creep, coalescence, and form a macrocrack, leading to fracture of the body [1].

In [2], with a description of brittle fracture, the parameter ω was introduced, i.e., the damage to the material, obeying the equation

$$\frac{\partial \omega}{\partial t} = A\left(\frac{\sigma_{\max}}{1-\omega}\right)^m \quad (0.1)$$

(A, m are constants of the material). For smooth samples, where the process of the generation and coalescence of the pores is uniformly distributed over the whole volume of the body, ω depends only on the time. In this case, Eq. (0.1) is easily integrated and gives a connection between the elongational stress σ_0 and the time up to fracture t_R:

$$t_R = \left[A(m+1)\sigma_0^m\right]^{-1}. \quad (0.2)$$

The presence of stress concentrators in the body leads to inhomogeneity of the field of the stresses and to localization of the fracture site. Similar problems have recently attracted more and more attention [3-5]. A large number of experiments have been made on investigation of the propagation of cracks from sharp notches in thin plates, and on the discovery of the parameters controlling the process of the development of cracks (see the review [6]).

The present article is devoted to a theoretical solution of the problem of the propagation of a crack in a thin plate under conditions of brittle fracture with creep.

1. We consider a thin plate with an initial straight crack with a length $2l_0$, which is elongated by the stress σ_0, applied at infinity orthogonal to the direction of the crack. We assume that the plate is in a state of creep, and that the applied stress σ_0 is less than the critical stress $\sigma_* = K_c/\sqrt{\pi l_0}$ in the analogous Griffiths problem. Near the tip of the crack there is a narrow plastic zone, whose greatest dimension is d, and the maximal elongational stress is equal to the yield point for the stress σ_s. We denote by $l(t)$ the position of the tip of the crack; $\sigma_y(t, x_0)$ and $\omega(t, x_0)$ are the maximal elongational stress and the damage at the point x_0 of the x axis at the moment of time $t > 0$ (the x axis is directed along the crack; the y axis is orthogonal to it). Integrating Eq. (0.1), we obtain a relationship connecting the damage and the stress at the point x_0:

$$1-(1-\omega(t, x_0))^{m+1} = A(m+1)\int_0^t \sigma_y^m(\tau, x_0)\,d\tau. \quad (1.1)$$

The criterion of fracture is the condition $\omega(t, l(t)+d) = 1$, i.e., the damage at the end of the plastic zone of the crack, which, at the moment of time t, had a length $2l(t)$ equal

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to unity. We introduce the dimensionless quantities $\tau = t/t_R$, $\lambda = l/l_0$, $\rho = d/l_0$, $\sigma = \sigma_y/\sigma_0$, $\sigma_1 = \sigma_s/\sigma_0$. Taking account of the fracture criterion and of the relationship (0.2), from (1.1) there follows an integral equation for the dependence of the length of the crack λ on the time τ

$$1 = \int_0^\tau \sigma^m(\tau_1, \lambda(\tau) + \rho)\, d\tau_1. \tag{1.2}$$

From Eq. (1.2) the elongation time of the crack $\tau_* = \sigma_1^{-m}$ is immediately determined. This is the time, during the course of which, at the end of the plastic zone of the initial immobile crack a damage accumulates equal to unity, and the crack starts to propagate. We rewrite (1.2) in the form

$$1 - \int_0^{\tau_*} \sigma^m(\tau_1, \lambda(\tau) + \rho)\, d\tau_1 = \int_{\tau_*}^{\tau} \sigma^m(\tau_1, \lambda(\tau) + \rho)\, d\tau_1. \tag{1.3}$$

With $0 \le \tau < \tau_*$, the crack is motionless, and the stresses change with the course of time only due to the process of creep. With $\tau > \tau_*$, the crack starts to propagate and the dependence of the stresses on the time is now determined not only by the creep, but by the non-steady-state character of the problem. An exact solution of Eq. (1.3) is complicated, and is impossible without the use of numerical methods (a similar problem with a different criterion of fracture was solved in [7] using the method of finite elements). However, the limiting states can be separated, and an approximate solution can be found to Eq. (1.3). Since the process of the accumulation of damage due to a strong concentration of stresses is localized near the tip of the crack, then, in the expression for the stresses, Eq. (1.3), only a single term can remain. We distinguish two classes of materials: "brittle" or "rigid," which are characterized by a high yield point and a small creep index and a power law of the creep, and "viscous" or "soft," for which the yield point is not great, and the creep index is rather large. For the first class of materials, τ_* is small; therefore, redistribution of the stresses due to creep can be neglected with $0 \le \tau \le \tau_*$, and it can be assumed that, in this interval of time, the stresses coincide with the initial elastic stresses. We postulate also that, also with $\tau > \tau_*$, the stresses vary according to an elastic law (this is true at least in the initial moment of the propagation of the crack). Consequently, for this class, $\sigma_y = K/\sqrt{2\pi r}$, where $r = x - l$ is the distance from the tip of the crack; $K = \sigma_0\sqrt{\pi l}$ is the coefficient of the intensity of the stresses. For the second class of materials, the redistribution of the stresses with $0 \le \tau \le \tau_*$ also cannot be neglected. After this time, in the plate let there be complete redistribution of the stresses from the initial elastic state to the state of fully established creep. In the case of a power law of the creep $\dot{\varepsilon}_{ij} = (3/2)B\sigma^{n-1}s_{ij}$, where B, n are constants of the material; $\sigma = (3/2 s_{ij}s_{ij})^{1/2}$ is the intensity of the stresses; s_{ij}, $\dot{\varepsilon}_{ij}$ are the components of the deviator of the tensor of the stresses and the deformation rates; for the elongational stress $\sigma_y = k_n\sigma_0(l/r)^{1/(n+1)}$ [8], where k_n is a known function of the creep index n ($k_1 = 1/\sqrt{2}$). We assume that, with $0 \le \tau \le \tau_*$, the stresses are determined from the state of fully established creep. Then, as for the other class of materials, we can write

$$\sigma_y = k_n\sigma_0(l/r)^{1/(n+1)}, \tag{1.4}$$

where n = 1 for "rigid" materials, and n > 1 for "soft" materials.

2. Taking account of expression (1.3), we rewrite expression (1.4) in the form

$$1 - \tau_* k_n^m (1/(\lambda(\tau) + \rho - 1))^{m/(n+1)} = k_n^m \int_{\tau_*}^{\tau} (\lambda(\tau_1)/(\lambda(\tau) + \rho - \lambda(\tau_1)))^{m/(n+1)}\, d\tau_1. \tag{2.1}$$

We make the replacement of variables $\lambda(\tau) = z$, $\lambda(\tau_1) = \xi$, $\tau_1 = \varphi(\xi)$, $d\tau_1 = \varphi'(\xi)d\xi$ and denote $m/(n + 1) = \alpha$ in the new variables, Eq. (2.1) will be

$$1-\tau_* k_n^m (z+\rho-1)^{-\alpha} = k_n^m \int_1^z (\xi/(z+\rho-\xi))^\alpha \varphi'(\xi)\, d\xi. \quad (2.2)$$

Equation (2.2) is a Volterra equation of the first kind with a difference kernel with respect to the function $\xi^\alpha \varphi'(\xi)$. The solution of this equation can be found using the Laplace transform $\bar{f}(p) = \int_0^\infty f(t) e^{-pt} dt$ (bringing Eq. (2.2) into standard form with a lower limit equal to zero). For the transformed quantities, Eq. (2.2) is written in the form

$$\frac{1}{q} - q^{\alpha-1} e^q \Gamma(1-\alpha, q) = k_n^m \rho^{-\alpha} \overline{\Phi}(p) q^{\alpha-1} e^q \Gamma(1-\alpha, q), \quad (2.3)$$

where $\Gamma(1-\alpha, q) = \int_q^\infty e^{-t} t^{-\alpha} dt$ is an incomplete gamma-function; $\overline{\Phi}(p)$ is a Laplace transform of the function $\xi^\alpha \varphi'(\xi)$; $q = \rho p$; it is also taken into consideration that $\tau_* = k_n^{-m} \rho^\alpha$ (which follows from Eq. (2.2) with $z = 1$). From Eq. (2.3) we find

$$\overline{\Phi}(p) = k_n^{-m} \rho^\alpha (q^{-\alpha} e^{-q} / \Gamma(1-\alpha, q) - 1). \quad (2.4)$$

Since the solution constructed is valid only with values of τ close to τ_* (i.e., $z \to 1$, and, consequently, $q \to \infty$), then, expression (2.4) can be replaced by an asymptotic expression; replacing $\Gamma(1-\alpha, q)$ by its asymptotic expansion with large values of q [9].

$$\Gamma(1-\alpha, q) = q^{-\alpha} e^{-q} (1 - \alpha/q + \alpha(\alpha+1)/q^2 - \ldots).$$

The final solution in transforms assumes the form

$$\overline{\Phi}(p) = k_n^{-m} \rho^\alpha ((1 - \alpha/q + \alpha(\alpha+1)/q^2 - \ldots)^{-1} - 1) \approx k_n^{-m} \rho^{\alpha-1} \alpha / p.$$

The inverse of this transform will be $z^\alpha \varphi'(z) = k_n^{-m} \rho^{\alpha-1} \alpha$. Returning to the old variables $\lambda = z$ and $\tau = \varphi(z)$, for the rate of propagation of the crack $d\lambda/d\tau$ we obtain the following dependence:

$$\frac{d\lambda}{d\tau} = k_n^m \frac{\rho}{\alpha} \left(\frac{\lambda}{\rho}\right)^\alpha,$$

or, in dimensional quantities (taking account of (2.2))

$$\frac{dl}{dt} = k_n^m \frac{A(m+1)}{\alpha} d^{1-\alpha} \sigma_0^m l^\alpha. \quad (2.5)$$

For "rigid" materials, $n = 1$, $k_1 = 1/\sqrt{2}$; consequently, (2.5) is written in the form

$$\frac{dl}{dt} = \frac{A(m+1)}{\pi m} (2\pi d)^{1-m/2} K^m.$$

A power dependence of the rate of propagation of a crack dl/dt on the coefficient of the intensity of the stresses K has been observed in a number of experiments [10, 11] (see, also the review [6]) for Cr—Mo—V steel.

In [12], for a description of the process of the propagation of a crack with creep, use was made of a modified Rice—Cherepanov J-integral $C^* = \oint \left[W dy - T_i \frac{\partial \dot{u}_i}{\partial x} ds\right]$, $W = \int_0^{\dot{\varepsilon}_{ij}} \sigma_{ij} d\dot{\varepsilon}_{ij}$, carried over to the case of fully established creep. For the power law of the creep, in accordance with [8], the expression of the components of the stress tensor in terms of C* has the form

$$\sigma_{ij} = \left(\frac{C^*}{B I_n r}\right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta), \quad (2.6)$$

where B, n are constants of the material in the power law; $\tilde{\sigma}_{ij}(\theta)$ are bounded functions of the polar angle θ; I_n is a constant depending on the index of the creep n. Taking account of (2.6) and (1.4), relationship (2.5) for a "soft" material is written in the form

$$\frac{dl}{dt} = A(m+1)\frac{n+1}{m} d^{1-\frac{m}{n+1}} (BI_n)^{-\frac{m}{n+1}} C^{*\frac{m}{n+1}}.$$

Consequently, for "soft" materials, a power dependence of dl/dt on C^* is obtained, which has been observed experimentally in [12] for nickel alloys. Thus, the following conclusions can be drawn: the kinetic equation for the parameter of the damage ω can be used also to describe the process of brittle fracture with the creep of samples with notches; here, the rate of propagation of cracks for "rigid" materials is determined by the coefficient of the intensity of the stresses, and, for "soft" materials, by the value of the modified J-integral.

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